Kinematics
Projectile Motion

CONCEPT EXPLORATION

You have investigated the motion of objects that followed a straight-line path. That kind of motion is **one-dimensional**. You have also considered the motion of objects that followed two separate straight-line paths that were perpendicular to each other. That kind of motion is **two-dimensional.**

Now you will investigate another kind of two-dimensional motion; the motion of an object that is thrown in the horizontal direction but begins to fall vertically to the ground under the influence of gravity. What sort of path does such an object follow before it strikes the ground?

Engagement Questions

1. The dart shown below is thrown in the horizontal direction from a position above the ground. Draw a picture of the path that you would expect the dart to follow as it flies through the air before it strikes the ground.

A **projectile** is an object that moves under the influence of gravity alone.

2. Using the definition given above, indicate which of the following objects would be considered projectiles?

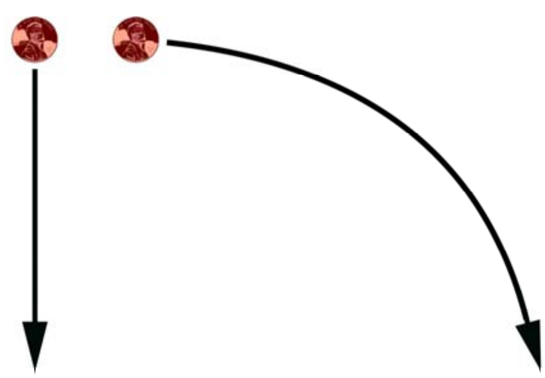
a. a flying bird
b. a flying paper airplane
c. a descending parachute
d. a thrown Frisbee
e. a bowling ball rolling across the floor
f. a bowling ball thrown horizontally off of a cliff
g. a bowling ball dropped off of a cliff

The Challenge

You will determine which of two projectiles hits the ground first; a penny dropped from a particular height or a penny thrown horizontally from the same height at the same time.

Your Ideas about the Challenge

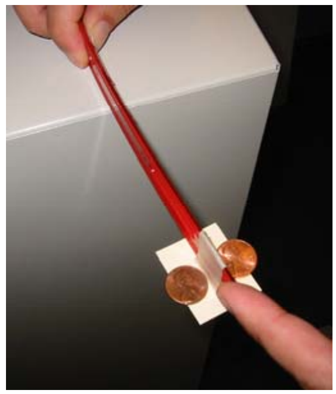
3. A penny is dropped from a height of 1 meter at the same time that another penny slides off of a counter top that is 1 meter tall. Which penny do you think will hit the floor first?



At each lab station you will find the following:
A plastic ruler device, two pennies

The Investigation

a. Place the plastic ruler assembly so that the ruler lies horizontally on its edge on the counter top with the paper platform lying flush with the counter.
b. Place the two pennies on either side of the paper platform so that they will be supported by the platform.
c. Keeping the ruler edge in contact with the counter top, carefully slide the platform-end of the ruler perpendicularly off of the counter so that the platform-end of the ruler extends over the edge of the counter approximately 15 cm.
d. Slightly bend the ruler back approximately 2 cm, being careful to keep the pennies in place on their platform.
e. Release the ruler so that one penny is projected out horizontally while the other penny drops straight down to the floor.
f. Repeat this process 10 times keeping track of which penny strikes the floor first.



4. Did either penny strike the floor first every time? If so, which one?

5. Was there a big time difference between the moments that each penny struck the floor?

6. What made each of the pennies strike the ground?

7. Evaluate the following student statements about the investigation you performed. Identify ideas that are consistent with your observations and others that are not consistent with your observations.

<u>Student A</u>
"The penny that dropped straight down should hit the ground first since it follows a shorter path".

<u>Student B</u>
"Both pennies should hit the ground at the same time since gravity gives all objects the same downward acceleration in the absence of air resistance."

Check your work with your teacher. ☐

8. A football is thrown from a height of two meters and is released in a direction that is parallel to the ground. Draw the shape of the path that the football would follow from the moment of release until it strikes the ground.

9. What makes the football strike the ground?

10. Does the football accelerate as it falls to the ground and if so, what is the direction for the acceleration?

11. What should be the rate of acceleration for this football as it falls to the ground? Be sure to include the appropriate acceleration units.

12. While the football is flying through the air what, if anything, would act on the football to change its horizontal motion? ⟷

13. If we ignore the effects of air resistance what should be the change in velocity (acceleration) of the football in the horizontal direction?

14. Evaluate the following student statements about the investigation you performed. Identify ideas that are consistent with your observations and others that are not consistent with your observations.

Student A

"An object that is thrown really hard in the horizontal direction will stay in the air for a longer time because it has lots of momentum".

Student B

"Dude, the horizontal motion of a projectile has no effect on the vertical motion. Gravity makes the projectile follow the curved path that makes it hit the ground, and gravity acts only in the vertical direction."

Check your work with your teacher. ☐

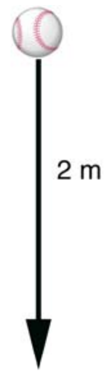
Kinematics
Projectile Motion

CONCEPT DEVELOPMENT

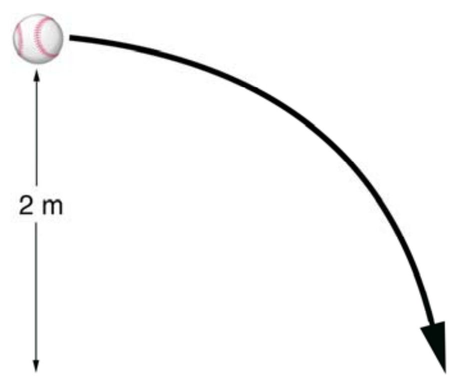
In the exploration activity you determined that all projectiles, in the absence of air resistance, experience the same downward gravitational acceleration. As you already know, gravitational acceleration close to the surface of the earth, is approximately $9.8\ \frac{m}{s^2}$. Using this quantity along with the one-dimensional kinematics equations you can predict the horizontal distance that a projectile would fly before it strikes the ground.

Engagement Question

1. A baseball is released from rest from a height of 2 meters. How long does it take the baseball to strike the ground?



2. This same baseball is hit with a bat so that it initially flies off of the bat in the horizontal direction. How long does it take the baseball to first strike the ground if it is hit at a height of 2 meters above the ground?



The Challenge

A ball bearing rolls off of a table and strikes the floor below. You will determine the horizontal distance that the ball bearing travels before it strikes the floor.

Your Ideas about the Challenge

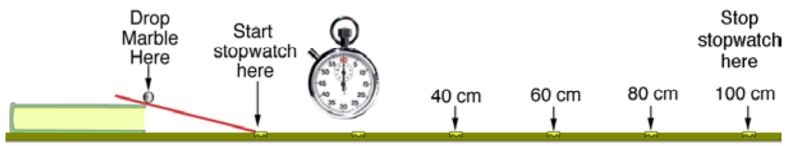
3. A ball bearing travels a distance of 1 meter in 0.8 seconds. Use the distance-rate-time equation to determine how fast the ball was rolling across the table. Be sure to label the answer appropriately.

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{1\ \text{m}}{0.8\ \text{s}} =$$

At each lab station you will find the following:
A plastic ruler, a ball bearing, a stopwatch, and a length of masking tape

The Investigation

a. You will use the plastic ruler like a ramp.
b. Set up your ramp on the counter so that you have at least 1 meter of counter space from the bottom of your ramp.
c. Place a book under the far end of your ramp and secure your ramp in place with masking tape.
d. Measure the height from the counter top to the high end of your ramp. Make this measurement to the nearest tenth of a centimeter, and record this in the data section that follows.
e. Tape a meter stick on top of the counter top so that it starts from the bottom of your ramp and lies along the path that the ball bearing will follow as it rolls along the counter top.
f. One lab partner should release the ball bearing from rest at the top of the ramp so that it accelerates down the ramp.
g. Another lab partner should start the stopwatch when the ball bearing first reaches the counter top, and stop the stopwatch when the ball reaches the end of the meter stick.
8. Repeat the timing procedure multiple times until you have 10 reasonably close time values. Record these 10 time values in the data section that follows.



Data Tables

Ramp height (cm)

Roll Number	Time (s)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

4. Calculate the average time from the table above and record this value to the nearest 10th of a second below.

Average Time (s)

5. Calculate the average speed of your ball bearing by dividing 1 meter by your average time. Be sure to include the appropriate units.

$$\text{Average speed} = \frac{1\text{ m}}{\text{average time}} =$$

Check your work with your teacher.

Give your ball bearing and stopwatch to your teacher and continue with the rest of the investigation.

The Investigation (continued)

a. Set up your ramp so that the bottom end of your ramp is approximately 20 cm from the edge of your counter top. The ramp should be aimed so that it is perpendicular with respect to the edge of the counter top.
b. Make sure that your ramp is inclined at the same angle as in the earlier investigation. Use the measured distance above the counter top from the previous data to help you make sure.
c. Measure the vertical distance from the floor to the top of the counter. Record this distance in meters in the data table that follows. Record this distance to the nearest 100th of a meter.

Data

Height (h) of your counter top (cm)

6. Calculate how long it would take for the ball bearing to drop the distance (h), shown above, by using the kinematics equation, $d = \frac{1}{2}at^2$, solved for time. Your height h, recorded above, is the distance that should be used in this equation. Be sure to label your answer with the appropriate units.

$$d = h = \frac{1}{2}at^2$$

solved for t

$$t = \sqrt{\frac{2h}{a}} =$$

7. Calculate the horizontal distance that the ball bearing will travel by using the distance-rate-time equation. You will use the velocity you calculated for the ball bearing rolling on the table (for r) and the time for the ball bearing to drop to the floor in your calculations (for t).

$$d = r\,t =$$

Check your work with your teacher. ☐

The Investigation (continued)

a. Measure the horizontal distance that you calculated above along the floor from the spot on the floor directly beneath the point that your ball bearing will leave the counter top.
b. Place a piece of masking tape on the floor with an “x” marked on the tape at the point that you expect your ball bearing to strike the floor.
c. When you are ready your teacher will bring you a ball bearing and a target that will be placed on the floor, centered on your marked spot.
d. Release the ball bearing from the top of the ramp in the same way that you had when you were determining its rolling speed.
e. Measure the difference in the distance (in centimeters) between the actual landing spot of your ball bearing and your predicted location. Record this value below.

Data

Difference between actual landing spot and your predicted location (cm)	

8. Why do you think that the ball bearing didn’t go the exact distance that you measured on the floor?

Check your work with your teacher.

Kinematics
Projectile Motion

CONCEPT REFINEMENT

Review

In the concept exploration and development exercises we investigated the characteristics of projectiles. We have also determined that a projectile launched horizontally and a projectile dropped from the same height will reach the ground at the same time. We found that the vertical and horizontal motion of a projectile are "independent" of one another.

We have also seen that projectiles with initial horizontal velocity follow a curved path. The nature and shape of this path will be the focus for this concept refinement exercise.

An artillery shell is fired with an initial speed of 20 m/s in the horizontal direction from the top of a tall cliff.

1. What is the horizontal acceleration of the artillery shell after it leaves the cannon barrel and flies through the air?

2. What is the vertical acceleration of the artillery shell after it leaves the cannon barrel and flies through the air?

You can calculate both the horizontal and vertical position for this artillery shell by using the equation $d = v_0t + 1/2at^2$.

The horizontal form of the equation shown above is $d = v_0t$.

3. Why does the equation reduce to this form for the horizontal position?

The vertical form of the original equation is $d = 1/2at^2$.

4. Why does the equation reduce to this form for the vertical position?

5. Calculate both the horizontal and vertical positions for the first 5 seconds of flight. For example the horizontal position at 1 second would be 20 meters and the vertical position at 1 second would be 5 meters.

Horizontal: $d = v_0t = (20\ m/s)(1\ s) = 20\ m$

Vertical: $d = 1/2at^2 = 1/2(10\ m/s^2)(1\ s)^2 = 5\ m$

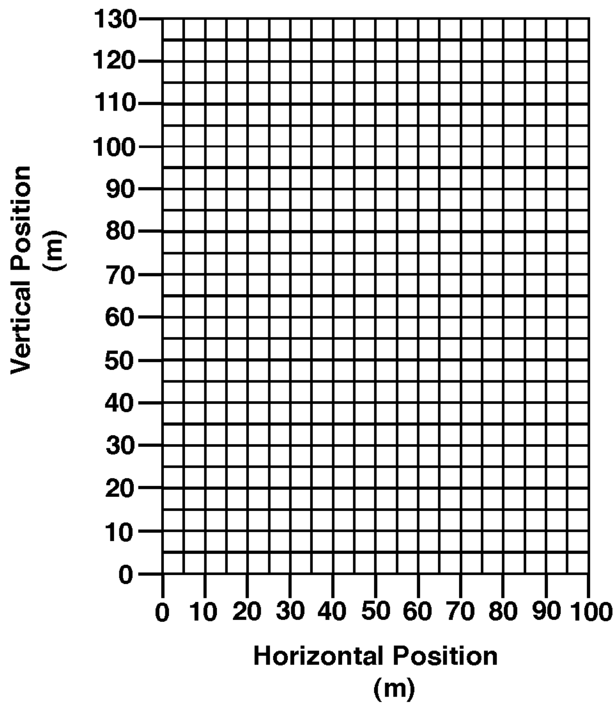
In the table, seen to the right, record both the horizontal and vertical positions for the first 5 seconds.

Start both the horizontal and vertical positions at zero and use +10 m/s^2 for the acceleration of gravity. This would make the downward direction the positive direction.

Time (seconds)	Horizontal Position (meters)	Vertical Position (meters)
0	0	0
1	20	5
2		
3		
4		

5		

6. On the position-position grid provided below plot the vertical position versus the horizontal position. In other words the vertical axis corresponds to the vertical position coordinates and the horizontal axis corresponds to the horizontal position coordinates.

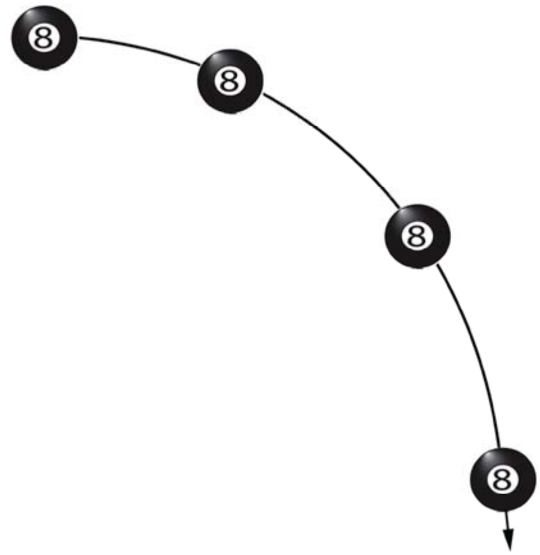


7. Draw a line that passes through the trend of the points.

8. Describe the shape of the resulting graph. Is it a straight line?

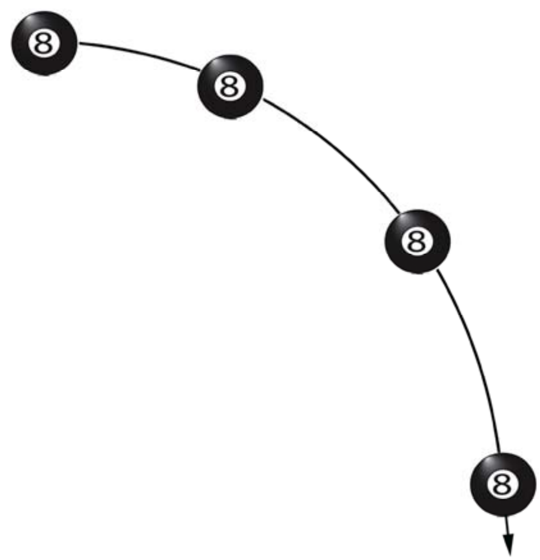
9. What is the name of the kind of curve that results when you graph a "squared" quantity. Hint: what shape do you get when you graph $y = x^2$.

Check your work with your teacher. ☐



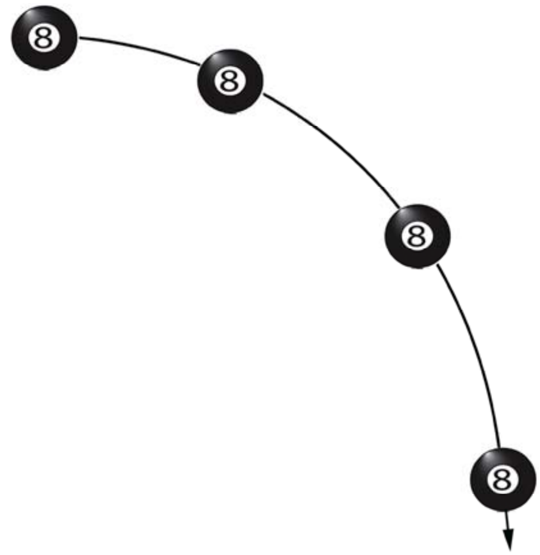
The curve shown to the right represents the path that a eight ball follows after it is thrown with an initial velocity of 20 m/s in the horizontal direction. Each picture represents the position of the eight ball at one-second intervals.

10. Draw vectors at each of the 4 positions shown that would represent what you think the horizontal component of velocity should look like at that particular position.



11. On the curve shown to the right draw vectors at each of the 4 positions that would represent what you think the vertical component of velocity should look like.

12. On the curve shown to the right draw the vector at each of the 4 positions that would represent the acceleration that the eight ball would experience as it flies through the air.



Check your work with your teacher. ☐